Chiral cosmic strings in supergravity

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Abstract

We consider F and D-term cosmic strings formed in supersymmetric theories. Supersymmetry is broken inside the string core, but restored outside. In global SUSY, this implies the existence of goldstino zero modes, and the string potentially carries fermionic currents. We show that these zero modes do not survive the coupling to gravity, due to the super Higgs mechanism. Therefore the superconductivity and chirality properties are different in global and local supersymmetry. For example, a string formed at the end of D-term inflation is chiral in supergravity but non-chiral in global SUSY.

1 Introduction

It is generically believed that the early universe went through a series of phase transitions associated with spontaneous symmetry breaking. This implies the existence of topological defects which form according to the Kibble mechanism [1]. Whereas monopoles and domain walls are cosmologically catastrophic, cosmic strings may have interesting cosmological properties [2].

An example of a phase transition occurring in the early universe is the phase transition triggering the end of hybrid inflation. Hybrid inflation can be embedded in grand unified theories (GUTs) [3]. In supersymmetric GUTs with standard hybrid inflation, all symmetry breaking patterns which are consistent with observations lead to the formation of cosmic strings at the end of inflation [4] (But see [5]). Both strings and inflation produce primordial density perturbations. The string contribution is constrained by the cosmic microwave background (CMB) data, which therefore puts strong constraints on models of hybrid inflation [6].[1]

Fermions coupling to the string forming Higgs field may be confined to the string [7]. The (massless) zero mode excitations give rise to currents along the string [8]. Of particular interest are “chiral strings”, strings which carry currents travelling in one direction only: the absence of current-current scattering enhances the stability of the chiral current. Not only do fermionic currents influence the evolution of the string network, and therefore alter the constraints from CMB data, they can also stabilise string loops against gravitational collapse [9]. The requirement that these stable loops, called vortons, do not overclose the universe also constrains the underlying physics.

Fermionic zero modes have been studied in the context of global supersymmetry (SUSY) [10, 11, 12]. The number of zero modes are given by index theorems [13], just as in the non-supersymmetric case. The new feature of SUSY theories is that SUSY is (partly) broken inside the string core. As a result a subset of the fermionic zero modes can be found by simply applying a supersymmetry transformation on the bosonic string background. In this paper we note that these zero modes are nothing but the goldstinos of broken supersymmetry. It is expected that when the theory is coupled to gravity these zero modes disappear, as the goldstino gets eaten in the super Higgs effect. As we will show, this is indeed the case. The result is that the zero mode spectrum is completely different, and therefore the properties of the string network, in global supersymmetry (SUSY) and in supergravity (SUGRA).

This paper is organised as follows. In the next section we give a review of the F and D strings, formed after symmetry breaking driven by F and D-terms respectively. SUSY is fully broken in the core of F-strings. There are then two zero modes, with opposite chirality, corresponding to the goldstinos. The D-string is a BPS solution.

[1]We note that predictions are always made based on numerical simulations of Nambu-Goto strings. They should be interpreted with care since reliable predictions can only be made using field theory simulations. Also remember that the evolution properties of supersymmetric string and of current carrying strings may actually differ from standard network simulations.

Only half of the SUSY is broken in its core, hence there is only one goldstino zero mode. In the absence of a super potential the string is chiral. If the winding number of the Higgs field is larger than unity, $n > 1$, the number of zero modes is multiplied by a factor n. We show how to find all goldstino modes in this case.

In section 3 we extend the results to SUGRA. Because of its BPS nature, the D-string is easiest to analyse. In particular we can evaluate the SUSY transformation on the bosonic background explicitly, to try to find the analogue of the goldstino zero mode. However, in SUGRA the zero mode (the gravitino component) cannot be confined to the string. This is a consequence of the super Higgs effect: the gravitino is massive inside the string and cannot be localised. The same is expected to happen for F-strings. We conclude that goldstino modes found in global SUSY are absent if the theory is extended to SUGRA.

In section 4 we look at the implications for the zero mode spectrum in the simplest realisations of F-term and D-term inflation. D-strings formed at the end of D-term inflation are non-chiral in global SUSY, as there are two zero modes with opposite chirality, the goldstino and one zero mode coming from the superpotential. In SUGRA the goldstino mode is absent, and D-strings are chiral. F-term strings are non-chiral in both SUSY and SUGRA. In the former case the only zero modes are the goldstinos, whereas in the latter there are no zero-modes. Coupling the Higgs to Majorana fermions will give a non-zero chiral current; such a Majorana current however is not expected to be persistent [22, 23].

We conclude in section 5.

2 F and D strings in global SUSY

We consider $N = 1$ supersymmetric $U(1)$ gauge theories. F-term inflation, and therefore F-strings, can also be embedded in non-Abelian theories or in $N = 2$ supersymmetric theories [11, 12]. Although details change in that case, our main conclusions concerning the goldstino zero modes remain the same.

Our conventions are the following. We use a Minkowski metric with $\eta_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$, and gamma matrices in the chiral representation

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix} \tag{1}$$

with $\sigma^\mu = (1, \sigma^i)$, $\bar{\sigma}^\mu = (1, -\sigma^i)$, and σ^i the Pauli matrices. For future convenience we also give the gamma matrices in cylindrical coordinates (t, z, r, θ): $\gamma^t = \gamma^0$, $\gamma^z = \gamma^3$, and

$$\gamma^r = \begin{pmatrix} 0 & \mathrm{e}^{-i\theta} \\ \mathrm{e}^{i\theta} & 0 \end{pmatrix}, \qquad \gamma^\theta = \begin{pmatrix} 0 & -i\mathrm{e}^{-i\theta} \\ i\mathrm{e}^{i\theta} & 0 \end{pmatrix}. \tag{2}$$

For spinors we use the conventions of Bailin & Love [14].

The $U(1)$ gauge theory contains an Abelian vector multiplet V, and an anomaly free combination of chiral super fields Φ_i with charges q_i. Including a Fayet-Iliopoulos (FI) term, the Lagrangian in component form reads

$$\mathcal{L} = \mathcal{L}_B + \mathcal{L}_F + \mathcal{L}_Y - U \ , \tag{3}$$

with

$$\begin{aligned}
\mathcal{L}_B &= (D^i_\mu \phi_i)^\dagger (D^{i\mu}\phi_i) - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} \ , \\
\mathcal{L}_F &= i\psi_i \sigma^\mu D^{i*}_\mu \bar{\psi}_i + i\lambda_i \sigma^\mu \partial_\mu \bar{\lambda}_i \ , \\
\mathcal{L}_Y &= ig\sqrt{2} q_i \phi_i^\dagger \psi_i \lambda - W_{ij}\psi_i\psi_j + (\text{c.c.}) \ , \\
U &= |F_i|^2 + \frac{1}{2}D^2 \\
&= |W_i|^2 + \frac{1}{2}(g\xi - g q_i \phi_i^\dagger \phi_i)^2 \ ,
\end{aligned} \tag{4}$$

where $D^i_\mu = \partial_\mu + igq_iA_\mu$, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, $W^{ij} = \delta^2 W/\delta\phi_i\delta\phi_j$ and $W^i = \delta W/\delta\phi_i = -F^{*i}$. The chiral fermions ψ_i and gaugino λ are left-handed Weyl fermions. They transform under an infinitesimal SUSY transformation as

$$\begin{aligned}
\delta\lambda &= -iD\epsilon - \frac{1}{2}\sigma^\mu\bar{\sigma}^\nu \epsilon F_{\mu\nu}, \\
\delta\psi_i &= \sqrt{2}\left(\epsilon F_i + iD_\mu\phi_i\sigma^\mu\bar{\epsilon}\right).
\end{aligned} \tag{5}$$

2.1 F-strings

Consider an Abelian theory without FI term, and a superpotential of the form

$$W = h\Phi_0(\Phi_+\Phi_- - v^2) \tag{6}$$

with $U(1)$ charges $\pm1, 0$ for $\Phi_\pm, \Phi_0$. This is the superpotential which leads to F-term hybrid inflation. [3]. Inflation takes place when ϕ_0 is slowly rolling down its potential, while $\phi_\pm = 0$ are fixed. It ends when the inflaton drops below a critical value $\phi_0 < v$, and the $U(1)$ breaking phase transition takes place. Cosmic strings are formed at the end of inflation through the Kibble mechanism [1].

The $U(1)$ breaking vacuum corresponds to $\phi_0 = 0$, $\phi_+\phi_- = v^2$ and $|\phi_+| = |\phi_-|$; this is the vacuum solution outside a cosmic string. In terms of Weyl spinors the Yukawa part of the Lagrangian reads

$$\mathcal{L}_y = -m_1\chi_1\xi_1 - m_2\chi_2\xi_2 + \text{h.c.}, \tag{7}$$

with $m_1 = 2gv$, $m_2 = \sqrt{2}hv$ and

$$\begin{aligned}
\chi_1 &= \tfrac{1}{\sqrt{2}}\left(-e^{-i\theta}\psi_+ + e^{+i\theta}\psi_-\right), \qquad & \xi_1 &= \lambda, \\
\chi_2 &= \tfrac{1}{\sqrt{2}}\left(e^{-i\theta}\psi_+ + e^{+i\theta}\psi_-\right), \qquad & \xi_2 &= \psi_0.
\end{aligned} \tag{8}$$

Here θ is the phase of the Higgs field, where we have written $\phi_\pm = v\mathrm{e}^{\pm i\theta}$. In the absence of the string the phase θ can be absorbed in the fields, and can be set consistently to zero. The fermions can be paired into two Dirac spinors

$$\Psi_i = \begin{pmatrix} \xi_{\alpha_i} \\ \bar{\chi}_i^{\dot{\alpha}} \end{pmatrix} \tag{9}$$

with $i = 1, 2$ and $\chi_\alpha = \epsilon_{\alpha\beta}\chi^\beta = \epsilon_{\alpha\beta}(\bar{\chi}^{\dot{\beta}})^*$.

Consider now a cosmic string background. The Nielsen-Olesen solution for an infinitely long string is [10, 15]

$$\begin{aligned} \phi_+ &= \phi_-^\dagger = v\mathrm{e}^{in\theta} f(r), \\ A_\theta &= -\frac{n}{g} a(r), \\ F_0 &= hv^2(1 - f(r)^2), \end{aligned} \tag{10}$$

and all other components zero. The winding number n is an integer. The profile functions f and a obey the equations

$$\begin{aligned} f'' + \frac{f'}{r} - n^2 \frac{(1-a^2)}{r^2} f &= h^2 v^2 (f^2 - 1) f, \\ a'' - \frac{a'}{r} &= -4g^2 v^2 (1-a) f^2, \end{aligned} \tag{11}$$

with boundary conditions $f(0) = a(0) = 0$ and $f(\infty) = a(\infty) = 1$. Supersymmetry is broken by the string background, but is restored on length scales $r \gg v^{-1}$ where the string forming fields approach their vacuum values.

The fermionic zero mode solutions solve the (r, θ) dependent part of the fermion equations of motion; they are solutions to the full (t, z, r, θ) dependent equations of motion with $E = 0$. Denote the zero mode by $\psi(r, \theta)$, which will be a linear superposition of the fermionic fields in the theory. For zero modes which are eigenfunctions of $\sigma^3\psi = \pm\psi$, solutions with non-zero energy E can be constructed: [7, 8]

$$\Psi_n(t, z, r, \theta) = \psi_n(r, \theta)\mathrm{e}^{\mp iE(t+nz)} \tag{12}$$

with $n = \pm$ the eigenvalues under σ_3, and the $\mp$ in the exponent corresponding to positive/negative frequency solutions. The $n = +$ mode is moving in the $-z$ direction along the string (left chiral), the $n = -$ mode travels in the $= z$ direction (right chiral). The dispersion relation is $E = k_z$, the modes travel at the speed of light.

To find the fermionic excitations of zero energy, the zero modes, we can also apply a SUSY transformation [10]. This is possible since a SUSY transformation leaves the energy of a given configuration unchanged. Moreover, starting with a static bosonic configuration, the fermionic excitations obtained in this way automatically satisfy the equations of motion (with $E = 0$). The fermionic excitation found by

applying a SUSY transformation is just the goldstino of broken global SUSY. Using Eq. (5) we find two independent zero mode solutions: [10]

$$
\begin{aligned}
\lambda_\pm &= \mp i \frac{na'}{gr} \epsilon_\pm, \\
(\psi_0)_\pm &= \sqrt{2} h v^2 \left(1 - f^2\right) \epsilon_\pm, \\
(\psi_+)_\pm &= \pm i \sqrt{2} v \left(f' \pm \frac{n}{r}(1-a) f\right) \mathrm{e}^{i(n\mp 1)\theta} \epsilon^*_\pm, \\
(\psi_-)_\pm &= \pm i \sqrt{2} v \left(f' \mp \frac{n}{r}(1-a) f\right) \mathrm{e}^{i(-n\mp 1)\theta} \epsilon^*_\pm,
\end{aligned} \tag{13}
$$

Here the spinor $\epsilon_\pm$ obeys

$$\sigma^3 \epsilon_\pm = \pm \epsilon_\pm. \tag{14}$$

Explicitly, $\epsilon_+ = \delta_+ \binom{1}{0}$ and $\epsilon_- = \delta_- \binom{0}{1}$, with $\delta_\pm$ a complex number. The plus mode is left chiral. It is a collective excitation of $\lambda, \psi_0, \psi_+, \psi_-$, or in terms of the vacuum fields a super position of the ξ_i, χ_i in Eq. (9). Likewise for the minus mode, which is a right chiral mode.

The zero modes correspond to the massless goldstinos (two chiralities) of broken SUSY. As SUSY is only broken inside the core of the string, the goldstino wave function drops rapidly outside. The number of zero modes can be inferred from index theorems [7, 13]: For each independent Yukawa coupling to ϕ there are $|n|$ fermionic zero modes, with n the winding number of the Higgs field, see Eq. (10). Likewise for each Yukawa coupling to ϕ^* there are $|n|$ zero modes, but with opposite chirality. Since for the F-string solution there are Yukawa couplings to both ϕ_+ and ϕ_-, i.e., to both ϕ and ϕ^*, there are zero-modes of both chiralities. For $|n| = 1$, the method of applying SUSY transformations gives all zero modes, given by Eq. (13).

2.2 Zero modes for $n > 1$

For $|n| > 1$, the method of SUSY transformations gives only one zero mode per vortex (anti-vortex) coupling, whereas the index theorem tells there are $|n|$. How to find these other solutions? To do so we note that the Lagrangian Eq. (4) has a larger symmetry than global SUSY. It is invariant under a local SUSY transformation with a transformation parameter ζ that satisfies

$$\gamma^\mu \partial_\mu \zeta(x) = 0. \tag{15}$$

This is because the variation of the global SUSY Lagrangian under a transformation with a local SUSY parameter is of the form $\delta\mathcal{L} = \partial_\mu \bar{\zeta} j^\mu = ...\partial_\mu \bar{\zeta}\gamma^\mu ... = 0$ by virtue of Eq. (15). Here the ellipses denote functions of the various fields in the theory. [2]

[2] In the Noether procedure to construct the local SUSY Lagrangian from the global one, the variation of the global Lagrangian is cancelled by adding a term $\mathcal{L} \ni a\bar{\psi}_\mu j^\mu$, with a a constant and ψ_μ the gravitino. Looking at the SUGRA Lagrangian the terms right-multiplying $\bar{\psi}_\mu$, i.e., the Noether currents, all have a factor γ^μ in there which can be brought to the left.

There are two independent solutions to Eq. (15)

$$\zeta_\pm = \mathrm{e}^{\pm im\theta} r^m \epsilon_\pm. \tag{16}$$

with m an integer and $\epsilon_\pm$ the projected spinors defined in Eq. (14). We can use the SUSY transformation with ζ to find a whole tower of fermionic zero mode ($E = 0$) solutions to the equations of motion. Only $|n|$ of them will be normalisable. This procedure gives all "higher" goldstino zero modes in accordance with the index theorem.

This can also seen by looking at the equations of motion. The equations split in two independent sets, one set involving the upper components of λ, ψ_i (eigenfunctions of σ^3 with positive eigenvalue), and one set involving the lower components (eigenfunctions of σ^3 with negative eigenvalue). The fermionic equations of motion for the first set, derived from the Lagrangian Eq. (4), are four coupled equations:

$$\mathrm{e}^{-i\theta}(\partial_r - \frac{i}{r}D_\theta)\lambda^* - g\sqrt{2}vf(\mathrm{e}^{in\theta}\psi_- - \mathrm{e}^{-in\theta}\psi_+) = 0, \tag{17}$$

$$\mathrm{e}^{-i\theta}(\partial_r - \frac{i}{r}D_\theta)\psi_0^* + ihvf(\mathrm{e}^{in\theta}\psi_- + \mathrm{e}^{-in\theta}\psi_+) = 0, \tag{18}$$

$$\mathrm{e}^{-i\theta}(\partial_r - \frac{i}{r}D_\theta)\psi_\pm^* + vf\mathrm{e}^{\mp in\theta}(ih\psi_0 \pm g\sqrt{2}\lambda) = 0. \tag{19}$$

To avoid notational cluttering we have omitted the subscript 1 from λ_1, ψ_{01}, $\psi_{\pm 1}$ to indicate upper spinor components. [3] The angular dependence can be removed by the substitutions

$$\begin{aligned} \lambda &= \tilde{\lambda}(r)\mathrm{e}^{i(l-1)\theta}, \\ \psi_0 &= \tilde{\psi}_0(r)\mathrm{e}^{i(l-1)\theta}, \\ \psi_\pm &= \tilde{\psi}_\pm(r)\mathrm{e}^{i(\pm n-l)\theta}. \end{aligned} \tag{20}$$

An analysis of the asymptotic behaviour of the equations tells that there are renormalisable solutions for $1 \le l \le |n|$ — this is the index theorem [7, 13]. The $l = 1$ solutions corresponds the the solution found by the SUSY transformation with chiral parameters $\epsilon_\pm$. It can be checked explicitly, using the equations of motion for the profile functions Eq. (11), that the solutions Eq. (13) satisfy the equations of motion. The solutions for higher l can be obtained from the solution for $l = 1$

$$\begin{aligned} \lambda^l &= \mathrm{e}^{i(l-1)\theta} r^{(l-1)} \lambda^1, \\ \psi_0^l &= \mathrm{e}^{i(l-1)\theta} r^{(l-1)} \psi_0^1, \\ \psi_\pm^l &= \mathrm{e}^{i(l-1)\theta} r^{(l-1)} \psi_\pm^1 \end{aligned} \tag{21}$$

with the superscript $l = 1$ denoting the $l = 1$ solution of Eq. (13). The extra terms in the equation of motion, coming from $\gamma^\mu \partial_\mu(\mathrm{e}^{i(l-1)\theta} r^{(l-1)})$ cancel among each

[3] The corresponding equations for the lower spinor components are obtained by replacing $\mathrm{e}^{-i\theta}(\partial_r - \frac{i}{r}D_\theta) \to -\mathrm{e}^{i\theta}(\partial_r + \frac{i}{r}D_\theta)$.

other. The zero modes of Eq. (21) are exactly the zero mode solutions obtained by performing a SUSY transformation with ζ_+ as given in Eq. (16) with $m = l - 1$. Analogously, all right chiral modes, those involving the lower spinor components, can be found by applying a SUSY transformation with ζ_- with $m = l - 1$.

2.3 D-strings

Consider supersymmetry breaking with a FI term and two oppositely charged chiral fields $\phi_\pm$ with charges $q_\pm = \pm 1$. Both fields are needed to cancel anomalies. For now, we set the super potential to zero: $W = 0$. The bosonic potential is minimised for $\xi = |\phi_+|^2 - |\phi_-|^2$. The vacuum manifold is degenerate under $|\phi_i|^2 \to |\phi_i|^2 + c$, with c a real constant. The special point $|\phi_-| = 0$, $|\phi_+| = \sqrt{\xi}$ corresponds to the BPS solution, which conserves half of the supersymmetry, as we will now show.

Take the following bosonic background configuration:

$$\begin{aligned} \phi_+ &= \sqrt{\xi} e^{in\theta} f(r) \ , \\ A_\theta &= -\frac{1}{g} n a(r) \ , \\ D &= g\xi(1 - f(r)^2), \end{aligned} \tag{22}$$

and all other bosonic fields zero. The fermions transform under infinitesimal SUSY transformations as in Eq. (5). We are interested in a background solution which leaves part of the SUSY invariant. So we require $\delta\lambda = 0$, $\delta\psi_+ = 0$. ($\delta\psi_- = 0$ automatically for the background configuration Eq. (22)). This gives the BPS equations

$$\begin{aligned} F_{12} \mp D &= 0, \\ \left(\partial_r \mp \frac{i}{r}(\partial_\theta + igA_\theta)\right)\phi_+ &= 0, \end{aligned} \tag{23}$$

with $F_{12} = 1/rF_{r\theta} = 1/r\partial_r A_\theta$. Here the $\mp$ signs correspond to chiral transformations with $\epsilon_\pm$ defined in Eq. (14). Only the lower sign, corresponding to a SUSY transformation with ϵ_-, gives (for $n > 0$) a string solution with positive energy density [16]). [4] This then gives the BPS equations in terms of the profile functions f and a:

$$\begin{aligned} n\frac{a'}{r} &= g^2\xi(1 - f^2). \\ f' &= n\frac{(1-a)}{r} f, \end{aligned} \tag{24}$$

The BPS solution to Eq. (24) breaks half of the supersymmetry. The SUSY transformations in the string background are invariant under a transformation with ϵ_-,

[4] All equations are also valid for $n < 0$ if the substitution $n \to |n|$ and $\epsilon_+ \to \epsilon_-$ is made. From now on we will specialise to the $n > 0$ case.

but not with ϵ_+. The fermion transformation with ϵ_+ gives the goldstino mode, which is confined to the string. Explicitly:

$$
\begin{aligned}
\lambda(r,\theta) &= -i2g\xi(1-f^2)\epsilon_+, \\
\psi_+(r,\theta) &= i2\sqrt{2}\sqrt{\xi}\frac{n}{r}(1-a)f\mathrm{e}^{i(n-1)\theta}\epsilon_+^*.
\end{aligned} \tag{25}
$$

This corresponds to a left moving mode. In the absence of a superpotential the Yukawa Lagrangian reads

$$
\mathcal{L}_y = ig\sqrt{2}\phi_+^*\psi_+\lambda. \tag{26}
$$

It follows from the index theorem that the SUSY transformation gives all the zero modes for $|n|=1$. The $l=1$ mode can be obtained from a SUSY transformation with constant parameter $\epsilon_\pm$, whereas the $l>1$ can be obtained formally by a space dependent SUSY transformation with ζ_+ given by Eq. (16). Thus for $W=0$ and $\phi_-=0$ the string has $|n|$ fermionic zero modes, all moving in the same direction: the string is chiral.

As already noted before the vacuum manifold is degenerate under simultaneous shifts in $|\phi_+|^2$ and $|\phi_-|^2$. These correspond to physically distinct vacua as the mass of the gauge boson transforms $m_A^2 \to m_A^2 + c$. Only in the special point in moduli space $|\phi_+| = \sqrt{\xi}$, $|\psi_-| = 0$ is the string solution BPS. (It can be easily checked that $\delta\psi_- \neq 0$ if $\phi_- \neq 0$, and SUSY is broken completely).

Although the vacuum is degenerate, the BPS string solution is the solution of lowest energy. Any non-BPS string will back react on the vacuum, and the total energy, vacuum plus string, relaxes to the lowest state which corresponds to $\phi_- = 0$ and a BPS string. This is dubbed the vacuum selection effect in the literature [17]. Hence, FI gauge symmetry breaking in global SUSY leads to chiral strings in the absence of a superpotential.

2.4 Back reaction

We can study the back reaction of the current on the bosonic fields using the same method of SUSY transformations. For the bosonic fields in the vector multiplet, performing a SUSY transformation with parameter η gives

$$
\begin{aligned}
\delta\phi &= \sqrt{2}\eta\psi, \\
\delta F &= i\sqrt{2}\psi\sigma^\mu\bar{\eta}, \\
\delta V^\mu &= i(\eta\sigma^\mu\bar{\lambda} - \lambda\sigma^\mu\bar{\eta}), \\
\delta D &= -\partial_\mu V^\mu
\end{aligned} \tag{27}
$$

Denoting the upper component of a spinor by a subscript 1, the result of SUSY transformation with η_1 is

$$
\begin{aligned}
\delta\phi = \delta F &= 0, \\
\delta V^z = -\delta V^0 &= 2\mathrm{Im}(\lambda_1^*\eta_1), \\
\delta D &= 0.
\end{aligned} \tag{28}
$$

The last equality is automatically for the zero energy mode, but it also holds for modes with non-zero momentum, due to the formal equivalence $\partial_0 \leftrightarrow \partial_z$ for the solutions Eq. (12). These results agree with [18].

3 D strings in supergravity

In this section we extend the analysis of fermionic zero modes for D-strings to supergravity. We start with a discussion of the BPS string solution in the absence of a superpotential. We defer a discussion of the implications for D-term inflation, which has a non-zero superpotential, to the next section.

3.1 D-strings

The BPS solution also exists in supergravity Ref. ([16]). The local $\tilde{U}(1)$ transformation which leaves the FI-term invariant is a combination of the flat space gauge transformation and a super-Weyl transformation. It is a local R-symmetry, which induces non-zero R-charge for the fermions and the superpotential. We will list here only the final results, more details and explicit expressions can be found in Refs. ([16, 19]).

Analogously to the global SUSY case, we consider a theory with one chiral field Φ charged under a $\tilde{U}(1)$ (which is a local R symmetry), a minimal Kähler potential $K = \phi^*\phi$, minimal gauge function $f = 1$, and zero superpotential $W = 0$. The bosonic part of the Lagrangian is given by

$$e^{-1}\mathcal{L}_b = \frac{1}{2}M_{\rm pl}^2 R + (D_\mu\phi)^\dagger(D_\mu\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - V^D \tag{29}$$

with $V_D = \frac{1}{2}D^2$ the same as in global SUSY, see Eq. (4). The covariant derivatives on the scalar fields contain the gauge connection A_μ, just as in global SUSY. For vanishing F-terms, the fermionic Lagrangian containing only the gauginos and chiral fermions is up to the determinant of the vierbein of the same form as in global SUSY, but with the covariant derivatives now containing spin, gauge, and R-symmetry connections. Lagrangian terms specific to supergravity are the kinetic term for the gravitino, whose covariant derivative contains both a spin and R-connection, and the terms mixing the gravitino with the other fermions:

$$e^{-1}\mathcal{L}_{\rm mix} = -\frac{i}{2}D\bar{\psi}_{\mu L}\gamma^\mu\lambda + (\gamma^\rho D_\rho\phi_+)\bar{\psi}_{\mu L}\gamma^\mu\psi_+ + \text{h.c.} \tag{30}$$

The metric can be written in cylindrical coordinates as

$$\mathrm{d}s^2 = \mathrm{d}t^2 - \mathrm{d}z^2 - \mathrm{d}r^2 - C(r)^2\mathrm{d}\theta^2. \tag{31}$$

The bosonic background for ϕ_+ and A_μ is of the same as in the global SUSY case, Eq. (22), but with the profile functions obeying different BPS equations. The

fermions transform as in Eq. (5). In addition there is the transformation rule for the gravitino:

$$\delta\psi_{\mu L} = 2\left(\partial_\mu + \frac{1}{4}\omega_\mu^{ab}\gamma_{ab} + \frac{1}{2}iA_\mu^B\right)\epsilon, \tag{32}$$

with ω_μ^{ab} the spin connection, and A_μ^B the gravitino $\tilde{U}(1)$ connection. The condition $\delta\psi_{\mu L} = 0$ can only be satisfied for a SUSY parameter which only depends on θ. A globally well-behaved spinor parameter (killing spinor) is

$$\tilde{\epsilon}_\pm = \mathrm{e}^{\pm\frac{1}{2}i\theta}\epsilon_\pm \tag{33}$$

with $\epsilon_\pm$ constant spinors satisfying the chirality condition Eq. (14). The requirement $\delta\lambda = 0, \delta\psi = 0, \psi_{\mu L} = 0$ under a SUSY transformation with $\tilde{\epsilon}_\pm$, gives us the supergravity BPS equations

$$\begin{aligned} n\frac{a'}{C} &= \mp g^2\xi(1-f^2). && (34)\\ f' &= \mp n\frac{(1-a)}{C}f, && (35)\\ (1-C') &= \mp A_\theta^B && (36) \end{aligned}$$

As before, positivity of the string energy requires to take the lower sign. The solutions can be solved at the origin and at infinity [16]. At $r \to 0$

$$f = 0, \quad D = g\xi, \quad a = 0, \quad A_\theta^B = 0, \quad C'(0) = 1, \tag{37}$$

while at $r \to \infty$

$$f = \sqrt{\xi}, \quad D = 0, \quad a = 1, \quad A_\theta^B = \frac{n\xi}{M_{\mathrm{pl}}^2}, \quad C = r\left(1 - \frac{n\xi}{M_{\mathrm{pl}}^2}\right). \tag{38}$$

3.2 Zero modes

To get the zero mode, the analogue of the goldstino mode of the global SUSY case, we proceed in the same way and perform a SUSY transformation with $\tilde{\epsilon}_+$ [20]. This gives the same solution for λ and ψ as in Eq. (25) with the substitution $1/r \to 1/C(r)$, and $\epsilon \to \tilde{\epsilon}$:

$$\begin{aligned} \lambda(r,\theta) &= -i2g\xi(1-f^2)\mathrm{e}^{-i\frac{1}{2}\theta}\epsilon_+, \\ \psi_+(r,\theta) &= i2\sqrt{2}\sqrt{\xi}\frac{n}{C}(1-a)f\mathrm{e}^{i(n-1)\theta}\mathrm{e}^{i\frac{1}{2}\theta}\epsilon_+^*. && (39) \end{aligned}$$

In addition the θ-component of the gravitino transforms non-trivially:

$$\psi_{\theta L} = 2iA_\theta^B = \begin{cases} 0 & (r \to 0) \\ \frac{2in\xi}{M_{\mathrm{pl}}^2} & (r \to \infty) \end{cases} \tag{40}$$

The solution does not fall off at infinity, see Eq. (38), and the gravitino is not localised on the string. The static fermionic solution has $E = 0$ since both kinetic and potential energy vanish outside the string for a massless gravitino with constant wave function. However, any zero mode moving along the string with momentum k would imply infinite energy. Likewise, the modes for $l > 1$ lead to a gravitino wave function that does not fall off at infinity.

It can be verified explicitly that the zero modes are solutions to the equations of motion. The equation of motion for the upper component of the gaugino field for example is

$$\mathrm{e}^{-i\theta}\left(\partial_r - \frac{i}{C}D_\theta\right)\lambda^* + g\sqrt{2\xi}\mathrm{e}^{-in\theta}\psi_+ + \mathrm{e}^{-i\theta}\frac{D}{C}\psi^*_{\theta L} = 0. \tag{41}$$

The θ dependence cancels out for the zero mode solutions of Eqs. (39, 40). The terms $\partial_r\lambda^*$ together with the term involving ψ_+ cancel by the second BPS equation (35), just as in the global SUSY case. The covariant derivative is

$$D_\theta\lambda^* = \left(\partial_\theta - \frac{i}{2}C' + \frac{1}{2}A^B_\theta\right)\lambda^* = iA^B_\theta\lambda^*, \tag{42}$$

where the second equality follows from the third BPS equation (36). The C' term comes from the spin connection $\omega^{12}_\theta = -C'$ and A^B_θ is the R-connection. Plugging it all in the equation of motion becomes $A^\beta_\theta\lambda^* + D\psi^*_{\theta L} = 0$, which is indeed identically zero for the zero mode solutions Eqs. (39, 40).

It is not surprisingly that the gravitino cannot be confined to the string. In the string core, where both D and $\gamma^\rho D_\rho\phi_+$ are non-zero, the fermionic fields acquire mass terms through the couplings in $\mathcal{L}_{\rm mix}$, wee Eq. (30) . This is nothing but the super Higgs effect, the goldstino gets eaten by the gravitino which acquires a mass in the process. Note that the gravitino mass is maximum in the string core and zero outside. This is the opposite from the usual fermionic fields which couple to the Higgs field, which are massless in the core of the string, but massive outside. It can be understood that such fermions can get confined to the string as their energy is lowered in the string core. But this is not the case for the gravitino.

We conclude that the goldstino zero mode present in the global SUSY case has no equivalence in the SUGRA case. Without a superpotential the SUGRA string has no zero modes.

The SUGRA analysis has been done for the D-term string, as the BPS-nature simplifies the equations considerably. However, the super Higgs effect is generic to supersymmetry breaking. Therefore the goldstino zero modes present in global SUSY theories are absent in the SUGRA extension, independent of whether the symmetry is broken by D- or F-terms, of whether the theory has $N = 1$ or $N = 2$ symmetry, or whether the theory is Abelian or embedded in some non-Abelian set-up.

4 Hybrid inflation

In this section we count the number of zero modes for F- and D-strings formed at the end of Hybrid inflation, both in global SUSY and in SUGRA. We limit the discussion to the standard/minimal model of hybrid inflation.

4.1 D-term inflation

We first consider the globally supersymmetric theory. The superpotential in standard D-term inflation is

$$W = h\phi_0\phi_+\phi_-. \tag{43}$$

The inflaton ϕ_0 is neutral under $U(1)$, and the Higgs fields ϕ_+ and ϕ_- have opposite charges. The potential including the FI term is

$$V = h^2|\phi_0|^2|\phi_-|^2 + h^2|\phi_-|^2|\phi_+|^2 + h^2|\phi_+|^2|\phi_0|^2 + \frac{g^2}{2}(\xi - |\phi_+|^2 + |\phi_-|^2)^2 \tag{44}$$

It is minimised for $\phi_0 = \phi_- = 0$ and $\phi_+ = \sqrt{\xi}$. The degeneracy of the vacuum present in the absence of a super potential is lifted. Inflation happens for large initial inflaton VEVs, while ϕ_0 is slowly rolling down its potential with the Higgses fixed at $\phi_+ = \phi_- = 0$. It ends when the inflaton drops below the critical value $\phi_c = g\sqrt{\xi}/h$ and it becomes energetically favourable for ϕ_+ to develop a VEV. At the end of inflation, during the $U(1)$ breaking phase transition, cosmic D-strings form through the Kibble mechanism.

The Yukawa terms in the Lagrangian are of the form

$$\mathcal{L}_Y = ig\sqrt{2}\phi_+^*\psi_+\lambda - h\phi_+\psi_-\psi_0 + \text{h.c.} \tag{45}$$

Far away from the string this corresponds to two massive Dirac particles. Index theorems tell us that a Dirac fermion coupling to a anti-vortex gives $|n|$ left moving zero modes proportional to ϵ_+, whereas a coupling to a vortex gives $|n|$ right moving modes proportional to ϵ_-. The left moving modes are nothing but the goldstino modes found in section 2.3 by applying a supersymmetry transformation. The second term in $\mathcal{L}_Y$, which comes from the superpotential, gives in addition rise to $|n|$ right moving modes. Therefore the D-strings formed at the end of D-term inflation in the global supersymmetric theory are non-chiral, they have both left and right moving modes.

D-term inflation can be generalised to supergravity, see [19] for the explicit construction. The super potential is the same as in global SUSY [5], and the Yukawa interactions are the same as in the global SUSY case, Eq. (45), augmented by $\mathcal{L}_{\text{mix}}$

[5]Since the gravitino and gaugino carry a non-zero charge under under $\tilde{U}(1)$ the anomaly cancellation conditions are different in SUGRA. In particular, three neutral fields are needed to cancel anomalies. We will assume that only one of them, the inflaton ϕ_0, is coupled to the Higgs fields $\phi_\pm$.

of Eq. (30) which includes terms involving the gravitino. Although details change in the SUGRA case, the index theorem still applies. There are thus $|n|$ right-moving modes from the $\phi\psi_0\psi_-$ coupling. However, as discussed in section 3.2, the goldstinos are auxiliary fields, they get eaten by the gravitino, and the goldstino modes are absent in SUGRA. As a result the D-strings forming at the end of D-term inflation are chiral in the supergravity context. There is only a right moving zero mode, which is a superposition of ψ_0 and ψ_-.

4.2 F term inflation

The strings formed at the end of F-term inflation can carry currents, but these are non-chiral in the globally symmetric theory — see the discussion in section 2.1. There are two goldstino modes, one left and one right moving. We expect analogously to the D-string case, that the goldstino modes are absent in the supergravity version of the theory, due to the super Higgs mechanism. Therefore the supergravity strings formed at the end of F-term inflation are not current carrying.

The only way to get a chiral F-term string is to add a Yukawa couplings of the form [19].

$$W_1 = h_1 \phi_+ \chi^2_{-1/2} \quad \text{or} \quad W_2 = h_1 \frac{\phi_+^2 \chi_{-1}^2}{M_{\rm pl}} \tag{46}$$

Such vortex-fermion couplings give rise to a right-moving fermion zero modes [19]. The result is a chiral supergravity string.

In phenomenological models, F-term inflation is often embedded in grand unified theories. If the symmetry broken at the end of inflation is $U(1)_{B-L}$, $(B-L)$-cosmic strings form [21]. The superpotential includes a coupling between the string forming Higgs field and the right-handed Majorana neutrino is exactly of the form Eq. (46). [6]

Note, however, that the fermion field χ in Eq. (46) is a Majorana particle, i.e., a particle which is its own anti-particle. In this case the fermionic current cannot be protected by lepton number conservation, and consequently it is not expected to survive for string loops [22]. Moreover, for $B-L$ strings, neutrinos zero modes are destroyed after the electroweak phase transition [23].

5 Conclusions

We considered the spectrum of zero modes in both F-term and D-term inflation. In global SUSY (part of) the zero-modes can be obtained by performing a SUSY transformation on the bosonic back ground. These zero modes are nothing than the goldstino modes, corresponding to broken SUSY in the string core. The goldstino zero-mode does not survive the coupling to gravity, as it gets eaten by the gravitino in the process — the super Higgs effect. As a result, the number of zero-modes,

[6] If Φ_+ is an $SU(2)_R$ doublet, the right-handed neutrino gets its mass via a non-normalisable term.

and therefore the properties of the strings, differ in global SUSY and SUGRA. In particular, F-strings are current carrying in SUSY, but not in SUGRA. D-strings are non-chiral in SUSY, but chiral in SUGRA.

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